Sample Problem Sheet

Nicola Talbot

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1. Find the gradient of the unit circle $(x^2 + y^2 = 1)$.

   Solution: Differentiating with respect to x gives:

   $$
   \begin{aligned}
   2x + 2y\frac{dy}{dx} &= 0 \\
   \frac{dy}{dx} &= \frac{-2x}{2y} \\
   &= \frac{-x}{\sqrt{1-x^2}}.
   \end{aligned}
   $$

2. Differentiate w.r.t. x:
   $$ e^{xy} = 2x + y $$

   Solution: Differentiating both sides w.r.t. x:

   $$
   \begin{aligned}
   e^{xy}(1y + x\frac{dy}{dx}) &= 2 + \frac{dy}{dx} \\
   xe^{xy}\frac{dy}{dx} - \frac{dy}{dx} &= 2 - ye^{xy} \\
   \frac{dy}{dx}(xe^{xy} - 1) &= 2 - ye^{xy} \\
   \frac{dy}{dx} &= \frac{2 - ye^{xy}}{xe^{xy} - 1}
   \end{aligned}
   $$

3. Differentiate the following functions:

   (a) $f(x) = 4x^2 + x - 1$
   Solution: $f'(x) = 8x + 1$

   (b) $f'(x) = -2\cos(-2x)$